2. G. N. Dul'nev and N. N. Tarnovskii, Thermal Regimes of Electronic Equipment [in Russian], Energiya, Moscow (1971).
3. G. N. Dul'nev and B. V. Pol'shchikov, "Generalized thermal model of electronic equipment with high component density," Izv. Vyssh. Uchebn. Zaved., Priborostr., No. 10, 121-125 (1976).
4. A. A. Samarskii, Theory of Difference Systems [in Russian], Nauka, Moscow (1977).
5. A.V. Lykov, Handbook of Heat-Mass Exchange [in Russian], Energiya, Moscow (1978).
6. Yu. G. Volodin and V. V. Gasanova, "Computer method for calculating the thermal regime of communications equipment," Communications Technology [in Russian], Moscow (1977), pp. 62-69.

APPROXIMATING THE EFFECTS OF THERMAL PULSE ACTION ON A METAL BY GENERALIZING THE DIAGRAM OF PHASE-BOUNDARIES DISPLACEMENT

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A surface thermal pulse excites in a metal a wave of phase transformations whose mathematical description requires a formulation of strongly nonlinear thermophysical problems [1]. A machine solution of such problems is difficult because of the large volume of calculations, even for individual pulse modes, but the task becomes still more unwieldy when numerical data covering many metals over a wide range of pulse modes are needed.

In this respect it is important that the results of computer-aided numerical solution of those thermophysical problems can be analytically generalized and on this basis, as has been shown in an earlier study [2], a system of equations can be proposed which will approximate, within an acceptable degree of accuracy, the final one-dimensional displacement of phase boundaries in a metal due to action of a thermal pulse. This makes it possible to use those equations for obtaining extensive information about many metals and pulse modes without going through a numerical solution of the actual thermophysical problems, which naturally deserves to be carefully considered.

In the earlier study [2] there was proposed a method of using those equations for constructing the diagram of the final displacement of phase boundaries in a metal due to action of a surface thermal II-pulse with a given surface energy density W and a variable action time t. In this study the problem will be considered in broader terms, viz. constructing a generalized diagram of the final displacement of phase boundaries in a metal with both parameters W and t of a thermal pulse varied.

The generalized diagram will be calculated and constructed so that it will describe, with sufficient accuracy, the quantitative relation between the main parameters of a thermal pulse W, t and the main results of its action on a metal. The effect of a thermal pulse can, moreover, be characterized by displacement of the melting front $y_m = y_m(W, t)$ or by displacement of the evaporation front $y_e = y_e(W, t)$, or by the relative displacement of the evaporation front α

$$\alpha = \frac{y_e}{y_m}; \quad \alpha = \alpha(W, t); \quad 0 \leqslant \alpha \leqslant 1. \tag{1}$$

A search for the optimum variant of this generalized diagram has revealed that it is most expediently drawn in the form of the relation between four quantities

$$\Phi = \Phi(W, t, y_m, \alpha). \tag{2}$$

The main difficulty in calculations for the generalized diagram is related to the need to find the roots of transcendental equations, which requires appropriate numerical methods of solution. These authors have developed an algorithm of calculations for the complete generalized diagram realizable on a small computer. It is based on the diagram representing the re-

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lations $y_m = y_m(t)$ and $y_e = y_e(t)$ with fixed values of the parameter W and with a variable pulse duration t. The typical form of such a diagram is shown in Fig. 1. Its calculation and thorough physical analysis can be found in the earlier report [2].

The method of calculation for this diagram hinges on its characteristic points 0, 1, 2, 3, 4. Point 4 corresponds to the maximum pulse duration t_4 at which the melting front (melting isotherm) moves beyond the boundaries of the metal. Point 3 corresponds to the pulse duration t_3 at which the melting front moves to the maximum depth in the metal. The range of pulse durations $t_2 \leqslant t \leqslant t_3$ corresponds to the conditions under which evaporation of the metal is negligible. The durations t_4, t_3, and t_2 are determined by the relations

$$t_4 = \frac{1.27W^2}{aC_v^2(T_m^* - T_0)^2}; \quad t_3 = \frac{0.222W^2}{aC_v^2(T_m^* - T_0)^2}; \quad t_2 = \frac{10W^2}{ar_v^2 f^2}, \tag{3}$$

with the dimensionless function f

$$f = 1 + \frac{C_v}{r_v}\left[\frac{T_m}{\ln(v_0 r_v f t/W)} - T_0\right] \tag{4}$$

and are calculated for given W and t by the method of successive approximations, with a fast convergence from any initial approximation.

Point 1 corresponds to the conditions under which theinduced thermophysical process reaches the steady state at the end of the pulse. The corresponding duration t_1 can be found from the equation

$$f^2 - \frac{W^2}{t_1 a r_v^2} = 0 \tag{5}$$

by the method of half-division, assuming that $t_0 < t_1 < t_2$, where t_0 is the duration corresponding to point 0 on the diagram. The latter is, in turn, determined from the equation

$$t_0 = \frac{10W}{r_v v_0\left[1 + \frac{C_v}{r_v}\left(\frac{T_b}{\ln 10} - T_0\right)\right]} \tag{6}$$

and is associated with the conditions under which the steady-state velocity of the evaporation front remains one order of magnitude lower than the velocity of sound in the metal (higher velocities of the evaporation front will not be considered here).

After the characteristic points $t_0, \ldots, t_4$ have been determined, the calculations proceed according to the following algorithm.

In the $t \leqslant t_0$ range

$$y_e = \frac{WI(z)}{r_v f}; \quad z = \frac{2W^2}{af^2 r_v^2 t}; \tag{7}$$

$$I(z) = \operatorname{erf}(z) + \frac{1}{z\sqrt{\pi}}[\exp(-z^2) - 1]; \tag{8}$$

$$\operatorname{erf}(z) = 2\Phi u(z\sqrt{2}) - 1, \tag{9}$$

where $\Phi u(x)$ is the normal probability density distribution function [3]. This function is calculated according to a standard program.

In the $t_0 \leqslant t \leqslant t_1$ range

$$x_m^* = \frac{ar_v f t}{W} U(\lambda); \tag{10}$$

$$U(\lambda) = \frac{\ln(T_b/\ln\lambda) - T_0}{T_m^* - T_0}; \quad \lambda = \frac{v_0 r_v f t}{W}. \tag{11}$$

In the $t_2 \leqslant t \leqslant t_3$ range one finds $y_m \simeq x_m^*$ from the equation

$$\operatorname{ierfc}\left(\frac{x_m^*}{2\sqrt{at}}\right) = \frac{C_v(T_m^* - T_0)\sqrt{at}}{2W}, \tag{12}$$

$$\operatorname{ierfc}(\beta) = \frac{1}{\sqrt{\pi}}\exp(-\beta)^2 - \beta\operatorname{erfc}(\beta). \tag{13}$$

Equation (12) is solved by the method of half-division, $x_m^*(t_2)$ and $x_m^*(t_3)$ being the limits of the interval containing the root of this equation with

$$x_m^*(t_3) = \frac{0.45W}{C_v(T_m^* - T_0)}. \tag{14}$$

The method of finding x_m^* in the $t_3 \leqslant t \leqslant t_4$ range is analogous. The root of Eq. (12) is in this case contained within the interval between the limits $x_m^*(t_3)$ and $x_m^*(t_4)$. The values of y_m in the $t_1 \leqslant t \leqslant t_2$ range are found by the method of graphical interpolation.

This procedure for calculating an individual diagram for a fixed value of parameter W and a variable pulse duration t can be repeated for other fixed values of parameter W, which will make it possible to construct a composite diagram for all the values of W under consideration. It must be noted, however, that a semilog scale, very convenient for an individual diagram, becomes unsuitable for a composite diagram, especially when the values of W cover several orders of magnitude. An analysis relevant to this reveals a necessity of plotting each of the three quantities W, t, and y_m on the composite diagram to a log scale. When also the quantity α is now plotted on the composite diagram to a functional scale according to relation (1), then the ultimate goal of calculating and constructing the generalized diagram has been attained.

Typical examples of generalized diagrams for specific metals are shown in Fig. 2a, b, c. When drawn with the proper technique, they are sufficiently accurate and very convenient for an analysis of a complex thermophysical process induced in a metal.

These grains indicate that the generalized diagrams for different metals are similar in form but, with all other conditions the same, differ appreciably in the corresponding values of the parameters. They exhibit three distinct ranges: first range of predominant evaporation ($\alpha \simeq 1$), second range of evaporation and melting ($0 < \alpha < 1$), and third range of predominant melting ($\alpha \simeq 0$).

The generalized diagram depicts the quantitative relation between four quantities $\Phi = \Phi(W, t, y_m, \alpha)$. On this rests the possibility of easily solving a whole lot of complex problems with the aid of this diagram: $(W, t \to y_m, \alpha)$; $(W, y_m \to \alpha, t)$; $(W, \alpha \to y_m, t)$; $(y_m, t \to W, \alpha)$; $(y_m, \alpha \to W, t)$; $(\alpha, t \to W, y_m)$.

In the problem $(W, t \to y_m, \alpha)$ are to be determined the effects y_m and α of thermal pulse action on a metal, with the pulse parameters W and t given. The practical aspect of this problem emerges when the quality of pulse treatment of a given metal is to be predicted in cases where a generator of thermal pulses with known performance parameters will be used for such a treatment.

In the problem $(W, y_m \to \alpha, t)$ is given one parameter of a thermal pulse W and one parameter of its effect y_m. The problem involves determining the second pulse parameter t and the relative displacement of the evaporation front α. An analysis of the diagrams for this case reveals that a given magnitude of y_m at a given W is attainable in two ways, viz., with and without evaporation of the metal by thermal pulses of very different durations, respectively. In practice this problem is associated with problems where requirements are imposed simultaneously on the pulse generator (parameter W) and the metal treatment technology (parameter y_m).

In the problem $(W, \alpha \to y_m, t)$ are given the parameter W of a thermal pulse and the relative displacement of the evaporation front α. The quantities y_m and t are to be determined. This problem has a unique solution only when $\alpha > 0$. In order to avoid an indeterminate answer with regard to the duration of a thermal pulse for a purely liquefying action on the metal ($\alpha = 0$), it is necessary to stipulate y_m, i.e., essentially to solve the problem $(W, \alpha, y_m \to t)$.

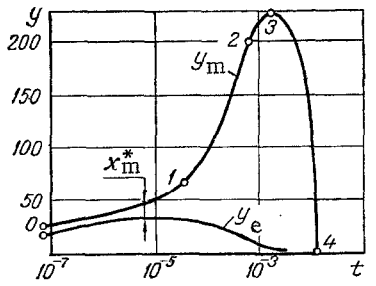

Fig. 1. Zinc: diagram of final displacements of phase boundaries y_e, y_m (μm), as functions of the pulse duration t (sec) at a fixed value of W = 1 J/mm², y (μm), t (sec).

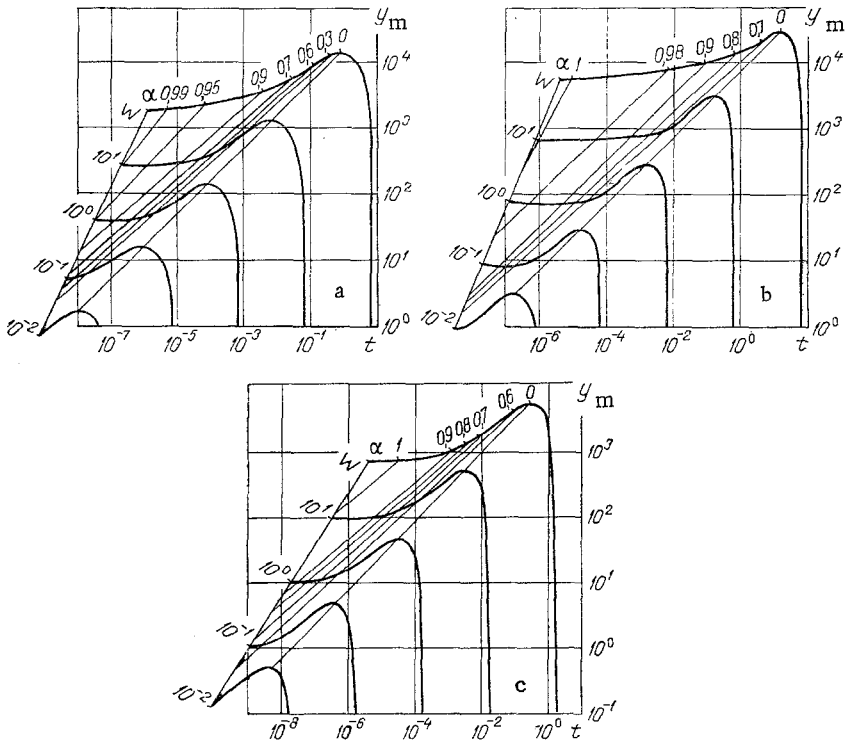

Fig. 2. Generalized diagram of final displacement of phase boundaries in (a) silver, (b) zinc, (c) tungsten; W (J/mm²), y_m (μm), t (sec).

It is well known that treatment of a metal with a thermal pulse can be effected in various ways. Erosion of a metal can occur preferentially either by the evaporation process or by the melting process. The second method (α = 0) is less suitable for attaining the necessary dimensional finish, while the first method (α ≈ 1) in contrast ensures just such a finish. Thus, also the practical significance of this problem becomes evident: it pinpoints the requirements which must be imposed on the pulse duration when the pulse parameter W and the mode of metal erosion have been specified beforehand.

In the problem (y_m, t → W, α) is given the displacement of the melting front y_m and the duration t of a thermal pulse, while the quantities W and α are to be determined. This problem has a unique solution for any given values of y_m and t. The practical significance of this problem is quite obvious. It pinpoints the requirements which must be imposed on the parameter W of a thermal pulse that a given pulse duration will ensure the needed displacement of the melting front y_m.

In the problem (y_m, α → W, t) are given the effects of a thermal pulse y_m and α, while the parameters of this pulse W and t are to be determined. When α > 0, then this problem has a unique solution. When α = 0, then the solution becomes a multivalued one. In order to avoid an indeterminate answer with regard to the pulse parameters, it is necessary to stipu-

late one of these parameters and solve the problem (y_m, α, $t \to W$) or (y_m, α, $W \to t$). The practical significance of this problem is obvious. Its solution actually determines the thermal pulse which will ensure the needed effects in a metal.

In the problem (α, $T \to W$, y_m) are given α and t, while W and y_m are to be determined. The solution is unique for $\alpha > 0$ and multivalued for $\alpha = 0$. The multivaluedness is removed by solving the problem (α, t, $W \to y_m$) or (α, t, $y_m \to W$).

The solution of all these problems, each formulated individually, involves problems of their physical simulation and is fraught with tremendous mathematical difficulties. The use of generalized diagrams, which are sources of easily read extensive information about the effects of thermal pulses in a metal, eliminates these difficulties and facilitates a quantitative analysis of the corresponding physical and technological problems.

The described method of constructing generalized diagrams is based on the one-dimensional approximation of a thermophysical process, which remains entirely valid as long as the zone of thermal influence in the metal is at least one order of magnitude wider than the displacement of the melting and evaporation fronts. From this condition follow some limitations on the applicability of the generalized diagram with respect to pulse modes. It is quite obvious that a pulse duration which will make the displacement of phase boundaries comparable with the width of the zone of thermal influence in the metal does not fit on the diagram. For each specific case one can, on this basis, estimate the particular limiting pulse modes and determine whether the generalized diagram is applicable.

NOTATION

C_V, specific heat at constant volume; r_V, heat of evaporation per unit volume, a, thermal diffusivity; T_m, melting point of a metal; T_m^*, referred melting point of a metal; T_0, initial temperature; v_0, velocity of sound in a metal; W, surface energy density of a thermal pulse, determined from the relation $W = Ft$, where F is the thermal flux density and t is the pulse duration; T_b, a characteristic temperature for a given metal, determined from the relation $kT_b = \varepsilon$, where k is the Boltzmann constant and ε is the atomic binding energy.

LITERATURE CITED

1. A. G. Goloveiko and A. D. Khudokormov, "Thermal wave in a metal with attendant phase transformations," Izv. Vyssh. Uchebn. Zaved., Energ., No. 3, 87-91 (1975).
2. A. G. Goloveiko and V. I. Martynikhina, "Excitation of a phase transformation wave in a metal by a surface thermal П-pulse," Inzh.-Fiz. Zh., 40, No. 4 (1981).
3. G. A. Korn and T. M. Korn, Mathematical Handbook for Scientists and Engineers, McGraw-Hill (1967).